9.13

$$E_{0R} = \frac{1-\beta}{1+\beta} E_{0I} \quad , \quad \beta = \frac{\mu_1 v_1}{\mu_2 v_2} \left(= \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \right)$$

$$R \equiv \left(\frac{E_{0R}}{E_{0I}} \right)^2 = \left(\frac{1-\beta}{1+\beta} \right)^2$$

$$E_{0T} = \left(\frac{2}{1+\beta} \right) E_{0I}$$

$$T \equiv \frac{I_T}{I_I} = \left(\frac{E_{0T}}{E_{0I}} \right)^2 \cdot \underbrace{\frac{\epsilon_2 v_2}{\epsilon_1 v_1}}_{\beta} = \frac{4}{(1+\beta)^2} \cdot \beta$$

$$R + T = \frac{(1-\beta)^2 + 4\beta}{(1+\beta)^2} = \frac{1 - 2\beta + \beta^2 + 4\beta^2}{(1+\beta)^2} = \frac{(1+\beta)^2}{(1+\beta)^2} = 1$$

Problem 9.14

Monday, April 19, 2010
6:32 PM

$$\tilde{E}_{0I}\,\hat{x} + \tilde{E}_{0R}\cdot\hat{n}_R = \tilde{E}_{0T}\,\hat{n}_T$$

FOR MAGN. FIELD:

$$\tilde{B}_{0I}\cdot\hat{y} - B_{0R}\cdot(\hat{z}\times\hat{n}_R) = \tilde{B}_{0T}(\hat{z}\times\hat{n}_T)$$

Y-component for E:

$$\tilde{E}_{0R}\cdot\sin\theta_R = \tilde{E}_{0T}\cdot\sin\theta_T$$

x-component for B:

$$\frac{\tilde{E}_{0R}}{\mu_1 v_1}\cdot\sin\theta_R = \frac{\tilde{E}_{0T}}{\mu_2 v_2}\sin\theta_T$$

This is possible ONLY if:

1) $\theta_T = \theta_R = 0$
(polarization is along $\hat{x}$)

or

2) $\mu_1 v_1 = \mu_2 v_2$ $\quad(\beta = 1)$

Usually possible only if
$v_1 = v_2$ $(\mu_1 \approx \mu_2)$,
$\rightarrow$ NO INTERFACE!

Problem 9.15

Monday, April 19, 2010
5:05 PM

$$Ae^{iax} + Be^{ibx} = Ce^{icx}$$

For $x=0$

$$\boxed{A+B=C}$$ (easy one!)

Take $\frac{\partial}{\partial x}$ of both sides:

$$iaAe^{iax} + ibBe^{ibx} = icCe^{icx}$$

For $x=0$

$$aA + bB = cC = c(A+B)$$

OR

$$(a-c)A + (b-c)B = 0 \qquad (1)$$

If we took $\frac{\partial}{\partial x}$ 2 times, we get:

$$a^2 A + b^2 B = c^2 C = c^2(A+B)$$

$$(a^2-c^2)A + (b^2-c^2)B = 0 \qquad (2)$$

From (1): $(a-c)A = (c-b)B$

(2): $(a-c)A\underline{(a+c)} = (c-b)B\underline{(c+b)}$

Since $A \neq 0$; $B \neq 0$

<u>either</u> $a-c = c-b = 0 \Rightarrow a=b=c$

<u>OR</u>: $a+c = b+c \Rightarrow a=b$

but then $aA + aB = cC = c(A+B)$

and $a = c$ $(=b)$

$\ast$ Another solution:

FIRST DERIVATIVE GIVES:

$$aAe^{iax} + bBe^{ibx} = cCe^{icx}$$

original eq. times a:

$$aAe^{iax} + aBe^{ibx} = aCe^{icx}$$

Subtract:

$$(b-a)B\cdot e^{ibx} = (c-a)\,Ce^{icx}$$

OR:

$$(b-a)B\cdot e^{i(b-c)x} = (c-a)\,C$$

left side depends on x, but right does not.

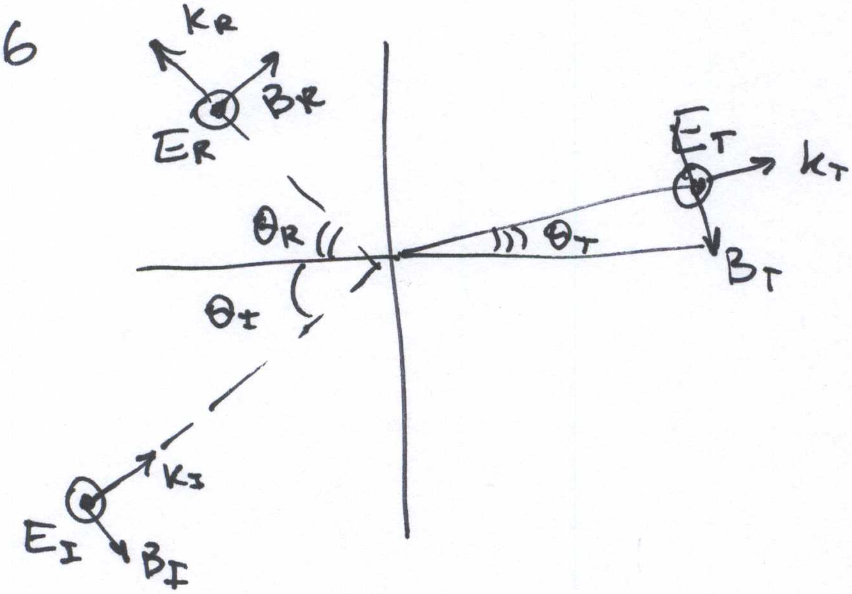
ONLY POSSIBLE IF $b-c=0$

OR $b-a=c-a=0$

Repeat with $\times b$ and $\times c$, shows $a=b=c$ and

$$A+B=C \quad (x=0)$$

9.16



BOUNDARY CONDITIONS:

(i) $D_\perp = \epsilon E_\perp$: $\quad 0=0$

(ii) $B_\perp$: $\quad B_{0I}\cdot\sin\theta_I + B_R\cdot\sin\theta_R = B_T\sin\theta_T$

OR, since $B=\frac{E}{v}$: $\quad E_{0I}+E_{0R} = \frac{v_1\cdot\sin\theta_T}{v_2\cdot\sin\theta_I}\cdot E_{0T}$

$\theta_R=\theta_I$

OR, since Snell's law $\frac{\sin\theta_T}{\sin\theta_I}=\frac{n_1}{n_2}=\frac{v_2}{v_1}$,

$$E_{0I}+E_{0R}=E_{0T}$$

(iii) $E_{\parallel}$: $\quad E_{0I}+E_{0R}=E_{0T}$ (SAME AS (ii))

(iv): $\quad \frac{1}{\mu_1}\left(-\frac{E_{0I}}{v_1}\cdot\cos\theta_I+\frac{E_{0R}}{v_2}\cdot\cos\theta_R\right)=\frac{1}{\mu_2}\cdot\left(-\frac{E_{0T}}{v_2}\right)\cdot\cos\theta_T$

$$E_{0I}-E_{0R}=E_{0T}\cdot\frac{\mu_1 v_1}{\mu_2 v_2}\cdot\frac{\cos\theta_T}{\cos\theta_I}$$

If as before, $\alpha\equiv\frac{\cos\theta_T}{\cos\theta_I}$, $\beta\equiv\frac{\mu_1 v_1}{\mu_2 v_2}\Rightarrow E_{0I}-E_{0R}=E_{0T}\cdot\alpha\beta$

9.16, cont'd: $E_{0T} = \frac{2}{1+\alpha\beta} E_{0I}$

$$E_{0R} = E_{0T} - E_{0I} = \left(\frac{2}{1+\alpha\beta} - 1\right) E_{0I} = \frac{1-\alpha\beta}{1+\alpha\beta} E_{0I}$$

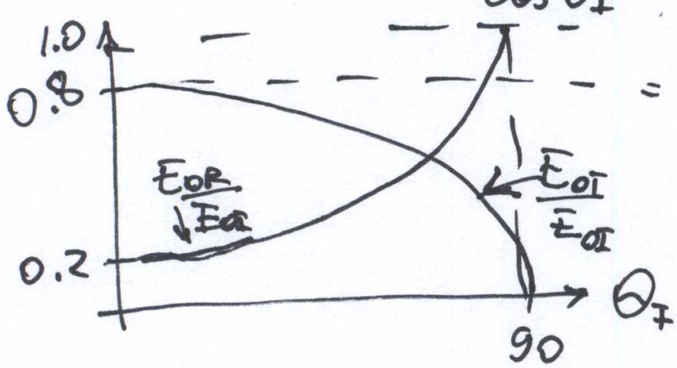
E_{0T} is always in-phase with E_{0I} $\left(\frac{2}{1+\alpha\beta} > 0\right)$

E_{0R} is in-phase with E_{0I} if $\alpha\beta < 1$
out-of-phase by 180° if $\alpha\beta > 1$

$$\alpha \cdot \beta = \beta \cdot \frac{\sqrt{1-\sin^2\theta_T}}{\cos\theta_I} = \beta \frac{\sqrt{1-\sin^2\theta_I \left(\frac{n_1}{n_2}\right)^2}}{\cos\theta_I}$$

or, since $\mu_1 \approx \mu_2$ $\quad \frac{n_1}{n_2} \approx \frac{1}{\beta}$

$$\alpha \cdot \beta = \beta \frac{\sqrt{1-\frac{\sin^2\theta_I}{\beta^2}}}{\cos\theta_I} = \frac{\sqrt{\beta^2 - \sin^2\theta}}{\cos\theta} > 0$$



Brewster Angle? $\quad E_{0R} = 0$ if $\alpha\beta = 1$

or $\quad \frac{\beta^2 - \sin^2\theta_I}{\cos\theta_I} \overset{?}{=} 1 \Rightarrow \beta^2 = \sin^2\theta_I + \cos^2\theta_I = 1$

But $\beta = 1.5 > 1$

For NORMAL INCIDENCE $\theta_I = 0$ $\quad \alpha = 1$ and

$$E_{0R} = \frac{1-\beta}{1+\beta} E_{0I} \qquad E_{0T} = \frac{2}{1+\beta} E_{0T}$$

$$R \equiv \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 \qquad T \equiv \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1} \cdot \alpha \left(\frac{E_{0T}}{E_{0I}}\right)^2 = \alpha\beta \left(\frac{2}{\alpha+\beta}\right)^2$$

$$R + T = \frac{1 - 2\alpha\beta + \alpha^2\beta^2 + 4\alpha\beta}{(\alpha+\beta)^2} = \frac{(\alpha+\beta)^2}{(\alpha+\beta)^2} = 1$$